Is Absolutely Everything Known Empirically? by Peter Gibson

Let us first distinguish between 'empirical' and 'empiricism'. Knowledge acquired through experience is said to be 'empirical', and science is the crowning glory of empirical knowledge. It is beyond dispute that a vast proportion of human knowledge is empirical. 'Empiricism', on the other hand, is a theory, and it is only worth discussing if it is expressed in a fairly strong form. The study of the subject suggests that this comes in two versions – either 'strong' empiricism, or 'very strong' empiricism. Locke tells us that all of our ideas come from experience (*Essay* 2.1.2), and Hume tells us that experience consists entirely of 'impressions', and that every idea in the mind derives ultimately from impressions (*Enquiry* 2.13). Empiricism is primarily a theory about how our concepts are formed, and this concept-formation offers a route from the external world to knowledge and thought. It is clear from their remarks that Locke and Hume count as at least 'strong' empiricists, because 'all' of our ideas are included.

It is always important, though, to realise that the experience invoked by empirical philosophers is not merely the experiences of the five senses, but also includes bodily awareness, emotions, and even the experience of thinking. The divide between the 'strong' and the 'very strong' concerns the status of the purest modes of thinking. If we think about very pure concepts like numbers or truth or the concept of existence, these seem far removed from daily experience, and one might surmise that they have some other source. Hence philosophers speak of some ideas being 'innate', or being known 'a priori', or arising through 'intuition' or through 'pure reason'. If a philosopher embraces such things enthusiastically, they are quickly expelled from the empiricist camp as traitors, and welcomed in by the rationalists. Even the most committed empiricist wavers in their faith, however, when faced with the unshakeable truths of simple logic, or that 7 + 5 = 12, since it is hard to see how daily experience could ever contradict such facts.

A solution to their problem was sketched by Hume, though, in the form of what he called 'relations of ideas'. Hume's thought was that we could hold unshakably to the key doctrine that all ideas derive from experience, but then allow that we have the capacity to compare the resulting ideas, and map their interactions. Hume must at least count as a 'strong' empiricist, because he offers a theory of how the very abstract areas of our thought can be rooted in experience. Hume's approach was the inspiration in the twentieth century for the logical positivists, and they took his division of our experience into 'relations of ideas' and 'matters of fact', but offered a version which was more precise, but also slightly different. Our knowledge falls into two classes, they said. Proper empirical knowledge comes from experience, gives us truths about the real world, and is entirely contingent in character. The other sort of knowledge, the descendant of Hume's relations of ideas, is labelled as 'a priori', 'analytic' and 'necessary'. The key term here is 'analytic', which we may define as 'true because of word meanings' (the classic instance being 'all bachelors are unmarried men'). This gives us a type of knowledge which is unashamedly non-empirical, but it is safely fenced off because it tells us nothing about the world at all, and is more like a game we play than a route to truth. A prodigious knowledge of chess openings would illustrate what they have in mind – it is knowledge, and is quite impressive, but is quite separate from nature. Mathematics and logic were held to fall into this category.

There is an important difference here between Hume's brief remarks and the logical positivist theory. Hume said that all of our ideas derive in some way from experience, even though the relations between them could become an independent study. The logical positivists, on the other hand, said that the ideas being studied could be brought into existence by our own definitions, and so could exist quite separately from any experience (in the manner that we decided how a knight will move in the game of chess). In this respect, then, we can say that the empiricism of Hume is 'stronger' than the empiricism of the logical positivist, because the dependence of all knowledge on experience is greater. Hume doesn't accept a 'fenced off' area of analytic truths, since the ideas being related all track back to experience. We can say that logical positivists are 'strong' empiricists, but Hume is 'very strong'. Given that Hume is a very strong empiricist, though, we might go on to ask whether his approach allows either a 'very strong' version, or a 'very very strong' version. If we treat every concept involved in our inner musings as rooted in experience, we are committed to 'very strong' empiricism. If we took a step further, though, we might become 'very very strong' empiricists by adding the claim that not only are the normal concepts of thought rooted in experience, but also that all the resulting higher level truths, and the further concepts that emerge in sophisticated activities like advanced mathematics and logic (concepts about concepts about concepts, perhaps), are also rooted in some way in experience. It is this 'very very strong' empiricism which I wish to examine. Is it possible that the doctrine of empiricism accounts for even the most rarefied and abstract truths which human endeavour has ever established?

To make such a claim we will have to deal with one obvious immediate objection, which is that it is fairly obvious that we are capable of inventing concepts like the knight-move in chess, and that we can invent any whimsical concept we like, give it a name, and draw inferences about it. As empiricists we can point out that the knight-move has to be built from experiences such as moving in steps, turning, and repetition, but no one thinks we have just observed the knight-move in nature. Even 'super-duper strong' empiricism in unable to deny that we can juggle with ideas in any way we like, and such frivolous jugglings just don't seem to be empirical. The plausible version of 'very very strong' empiricism will have to say, then, that there is a world of 'important' and 'serious' abstract knowledge which can be characterised as 'very high-level' experience. There will need to be a line drawn across the world of ideas, to separate the games and whimsy and idle speculation on one side, and the significant truths on the other. I won't explore that divide here, but the idea would be that mathematics, for example, is strikingly useful in science and technology, and Galileo famously said that the book of nature is written in mathematics. In the sort of instances where cutting edge geometry suddenly helps Einstein to formulate General Relativity, the 'very very strong' empiricist will say that this is the sort of serious mathematics that is wholly rooted in experience, and thus occasionally reveals the

principles of physical reality to us. Similarly, very high levels of logic might be embodied in a computer program, and (provided accurate data is the input) produce surprising complex truths to us, where the complex truths rest not just on the data, but also on the fact that the logic itself is based on the real world.

It is the logic that I wish to examine more closely in the context of empiricism. It would be nice if we could relegate whimsical and unreliable ideas to the harmless side of the line where the knight-move is located, and say that logic is to be found on the 'serious' side of the line, but anyone who has dabbled in logic will know that there is a profusion of competing systems in logic, and the idea that each system can appeal equally to the authority of underlying experience seems very implausible, given that the various systems only exist (like religious sects) by contradicting one another. Again, that is an issue which cannot be explored here, so my plan is to stick to fairly safe ground, by focusing on what is generally seen as the 'classical' version of the system known as 'propositional' or 'sentential' logic. Not only will I avoid the multitude of logical systems known as intuitionist, modal, many-valued, free, conditional, fuzzy and relevant logics, but I won't even consider classical first-order predicate logic (the one with quantifiers, objects and predicates). I will try to keep to shallow water, and have a quick look at the logic of how whole sentences relate to one another in their implications.

Before plunging into some detail, let me step back, and say something about the spirit of the present enquiry, and its context. If one wishes to actually prove the thesis that all the significant areas of high-level abstract thought are directly grounded in experience, it is hard to know where to start. To prove anything about logic is notoriously difficult, because the use of logic for the job implies instant circularity. The best that can be done is to offer a plausible picture, and suggest that it fits in with our wider views about a range of other matters. In a word, we might find a 'coherent' account of our knowledge if it was united into a single empiricist picture. The view that logic is thoroughly empirical has had very few champions. Aristotle, who seems to have invented logic by trying to encode the patterns of argument he was listening to in Plato's Academy, would probably have sympathised which such a unified approach, because, although as far as I know he does not directly address our question, he seemed to hope for a unified account of every area of study. The person who kicked the empirical view of logic far into the long grass was Frege, and for most modern students of the subject it has remained there. Frege has the enormous authority of being the inventor of quantified predicate logic, and he was passionately committed to a firm divorce of the truths of logic from anything to do with the mere human mind, with his pet hate being anything that smacked of 'psychology'. Frege defended what he called the 'third realm' of truths, where the first two realms concerned the mind, and the external world. The third realm contains what are now referred to as 'abstract objects', where 'abstract' means not existing in space or time, and being devoid of causal powers. This realm is one in which the truths of mathematics and logic are entirely self-sufficient, and have nothing to do with the physical world. In effect, Frege was a sophisticated modern platonist, and clearly a rationalist rather than an empiricist.

Nearly all students of modern logic seem to adopt either the Fregean or the logical positivist view of their subject. That is, they either think logic exists in its own abstract world, or they think it is simply human linguistic conventions, rules invented for either our convenience or our entertainment. Virtually no logician defends the view that logic might be empirical, so my musings receive almost no official support, with the one rather splendid exception of Bertrand Russell. In *The Problems of Philosophy* he asserted that the so-called Laws of Thought are beliefs about things, and not only beliefs about thoughts. For example, of the Law of Non-Contradiction, which he defines as "nothing can both be and not be", he observes that this law "is not the belief that if we *think* a certain tree is a beech, we cannot at the same time *think* that it is not a beech; it is the belief that if the tree *is* a beech, it cannot at the same time *be* not a beech" (p.50). The other two laws of thought are the Law of Identity – "whatever is, is" – and the Law of Excluded Middle – "everything must either be or not be" (p.40), and we can presume that Russell has similar views about those. They reflect facts about the world, not facts about thought, so that when something is a distinct individual it can't be some other individual as well, and also things either exist or don't exist, and either have some property or don't have it. Russell found that to be true of his experience, and so have most other people.

Russell was by no means a whole-hearted empiricist, since he was quite enthusiastic about a priori knowledge and the existence of universals, but in 1940 he went further in his surmises that logical thought grew out of experience. In Chapter 5 of *An Inquiry into Meaning and Truth* he works through the main connectives of classical logic, and tries to bring out the essence of each one. He doesn't launch into what I have called 'very very strong' empiricism, which would reduce all of the main components of logic directly to experience, but what is striking is the very un-Fregean way in which he embraces psychology to account for them. Of the word 'not' he observes the difficulty for an empiricist of knowing what experience corresponds to the proposition 'this is not white'. He then moves into a lengthy discussion of how the word 'or' can relate to our experiences. He again notes that we don't encounter 'or' directly in experience, because a road junction does not contain a left junction, a right junction, and a left-or-right junction. Hence his empirical account of 'or' bases it entirely in psychology, claiming that it reports mental states of questioning or dilemma. He defines 'or' as 'a verbal expression of indecision' (p.80).

With support from Russell for my thought that logic might be reducible to experience, even if the experience is psychological, I wondered whether the question might be explored a little more systematically. When I first attempted to study logic, I was strongly advised by two independent experts to use Lemmon's book *Beginning Logic*, but I was not told why. I later saw a passing remark that Lemmon sets basic logic out in the form known as 'natural deduction', and a little enquiry about that concept began to reveal the point to me. Natural deduction was an approach to logic devised by the great Nazi logician Gerhard Gentzen, and the idea was to build logic entirely from rules, and make no assumptions at all about what was true. The basis for a logic is a minimal number of logical connectives, and one or two initial rules; subsequent rules lay down when you can introduce one of the connectives into the argument, and

when you can eliminate one of the connectives. When an actual argument is constructed, every step of the argument is shown, and for each step one rule must be invoked. The beauty of natural deduction is that you then have a picture of logic in which there is a pure framework of reasoning which is independent of any truths we believe, and in which it is broken down into the smallest and clearest steps possible. The idea for this talk is that if we are to investigate the extent to which logic is empirical, natural deduction is exactly the format that is needed, because each building block of reasoning can be assessed independently. If we became convinced that every building block was rooted in experience, and if the truths we input into the argument structure also came from experience (which empiricists are inclined to believe), then this would be a potential triumph for hardline empiricism, and a difficult challenge for its opponents.

Before coming to the main point, it is worth observing that one might want to attempt similar empirical reductions of arithmetic and geometry. Aristotle offers a way to reduce arithmetic to experience, via the route of a theory of unity for each object, which can then be treated as a 'unit', which leads to the idea of counting. John Stuart Mill is a more recent defender of the empirical base for arithmetic, starting from the experience of looking at a group of pebbles (1843: 2.6.2). An obvious place to start for empirical geometry might be a critique of Plato's famous view in his dialogue *Meno* that a slave boy has innate geometrical proofs in his head, by asking what the boy has learned about spatial relations during his work. It should be noted, though, that mathematical reasoning is totally dependent on logic, so starting with the logic seems to get to the heart of the matter. A full reduction of all of our knowledge to an empirical base would, of course, have to include knowledge of general truths, knowledge of aesthetic and moral values, and anything else we can think of, but that is for another day. Let me finally, then, get to the point, which is whether the components of classical propositional logic which are laid out before us in natural deduction form might each of them be best understood as simply descriptions of experience.

[take a brief look at the handout on natural deduction]

My approach here is, I'm afraid, shockingly unphilosophical. I will speculate about the life of an early hunter-gatherer, who is presumed to be an intelligent and articulate homo sapiens living before the invention of writing. Beyond natural empathy for a member of my own species, I have no evidence for how such a person thought, but here goes anyway. Starting at the top of the natural deduction list, what would the introduction and elimination rules mean for such a person, in terms of experience? This person is walking alone along a forest path, trying to assess dangers. Their thinking has to be what we call 'counterfactual', of weighing up 'what-if' scenarios. If they assume a wild animal ahead, they decide to turn back. If they press on and find no animal, they give up the assumption. So rules of assumption obviously arise from experience.

The second row has the rules for 'and'. We've killed a deer and we've killed a swan. Individually that would be nice, but together ('deer *and* swan'!) we can have a feast! That introduces 'and'. Alternatively, we've killed a deer and a swan, so we can have swan for supper. That eliminates it.

This is obviously becoming boring, so I will scamper through the rest. I have given examples about Moore and Russell on the handout, which offer mere illustrations of the connectives, rather than claims about their origin, but obviously the situation at a philosophy conference has all the features of daily life that faced the hunter-gatherer. If you offer me a knife, then I have (as I had hoped) either a knife or a spear, so I've introduced 'or'. If we have a knife or a spear (somewhere in the luggage) and either one will kill the deer, then we can kill the deer, so that eliminates 'or'.

When we come to 'not' the so-called rules of introduction and elimination are a bit strained. Many logicians take negation as utterly primitive, but Bostock produces natural deduction rules for it, in order to show that the entire system *can* be presented in natural deduction form. I could contrive examples of my hunter-gathering using the rules, but let us just take denial of some proposition (such as 'there's an animal coming' 'no there isn't') as showing how basic negation is for us.

The two rules for the arrow which is usually characterised as 'if-then' are a little different from the other examples, because they are generally found as rules, rather than as connectives. Lemmon presents them as rules, and it is Bostock who shows that these rules can be reduced further, as ways of introducing and eliminating the arrow. Lemmon's rules are called 'conditional proof', and the familiar rule of 'modus ponens'. Skipping the details, to get a reductive empirical account of these two we just need to accept that implication is a natural experience. This would seem to be covered by the universal experience of evidence, for which footprints will do the job among hunter-gatherers. We introduce the arrow by saying that footprints always seem to mean animals are around, and we eliminate the arrow if we say footprints mean the presence of animals, and I've just seen footprints.

The use of 'not not' was probably not widespread in the Neolithic era, but a primitive grasp of 'not' should be enough for a pedantic hunter-gatherer to introduce and eliminate it. That completes my rather swift survey. I haven't proved anything, but I hope you see my point. Every single ingredient of classical logic has just been cited, and not one component takes us any distance from the most basic and simple experiences of daily living. Opponents of the empirical approach might reply either in the Fregean manner, that there is a platonic realm (Frege's 'third realm') which is parallel to the natural world but in tune with it (rather as Plato's Forms are linked to reality), or they can reply in the logical positivist manner, that the rules we have looked at have simply been invented by logicians because they helped in thinking about reality, but that further connectives could also be invented. There is a notorious connective called 'tonk', invented by Arthur Prior, to show that you could have a connective that proved anything you like (though one of the objections to 'tonk' is that no one can devise consistent natural deduction rules for it).

Two issues need further exploration. The first issue is the question raised by Russell, of whether these connectives are giving the structure of the real world, or the workings of psychology. In Russell's example about the Laws of Thought he connected logic to the real world of the beech tree, but in his later analysis of 'or' he connected it to psychology. Are the natural deduction rules just descriptions of how our minds work (on a good day), or are these logical concepts in some way a feature of the real world? The second issue is the role of language, since many of the hunter-gatherer thoughts I have proposed are within the grasp of a clever non-verbal animal. Is logical thinking only possible by means of language, and does language take us so far away from experience that the empiricist thesis is undermined? In both instances I am inclined to defend the realist view, rather than the psychological one.

On the first question (of realistic versus psychological interpretations), the main reason that Frege hated all reference to psychology when discussing the philosophy of logic was that our minds are riddled with falsehoods, and logic is only concerned with the communication of truth. For Frege, logic gives us an ideal of how we *should* think, not a description of the disappointing reality of thought. Clearly he has a point, and he offers a good reason for avoiding the psychological approach that Russell flirted with in 1940. But if we embrace the realist view that Russell offered about the beech tree in 1912, we have a different picture. If we say that given something is a beech tree it cannot fail to be a beech tree, that seems to be true. If I thought something was both a beech tree and an elephant, my friends would rightly call a doctor, because that is just *wrong*. That is, the truth which Frege demands is guaranteed by the facts of the world that is the subject of the reasoning. If the principles of the logic itself are fixed by the external world, that will provide the only ideal of truth transmission to which human beings have ever aspired, since it gets us from one truth about the world to another. Hence the realistic approach has the fixed stability which the psychological approach lacks.

That seems fine for the very basic Laws of Thought, but what about the logical connectives on the handout? Does it make any sense to say that 'and', 'or', 'not' and 'if-then' are features of the world, rather than features of our thought? Here I part company from my only ally, because I think that does make sense, and Russell was wrong. 'Or' was Russell's favoured example for his psychological approach. Consider a lizard living in an exceptionally hot environment, which can only survive if it spends some part of its day in the shade. Does it matter what provides the shade? No. It could be a rock, a tree, a cloud or another lizard. Any one of those will do, so the situation is what we call a 'disjunctive fact' – that the lizard will survive if it is shaded by *a* or *b* or *c* or *d*…. No human mind or language is required to fix this disjunctive fact. If one rock falls on a lizard it might survive, but if two fall on it it won't – that's a '*con*junctive fact'. If a falling rock hits the lizard it dies, but if it misses it survives – that is a 'conditional fact' – the physical embodiment of 'if-then', the arrow. If the lizard eats its survives; if it doesn't, it won't. Nothing is more vivid than 'not' in the life of a lizard. I am not claiming that the lizard reasons using logical connectives, but only that such things are embodied in the situations in which it finds itself. Lizards live in a world of disjunctive, conjunctive, conditional and negative facts, and they are central to any lizard's experience.

The second question I raised was whether language is essential to get proper logic going. There is a lovely counterexample offered by the Stoic philosopher Chrysippus, who wrote that 'A dog makes use of the fifth complex indemonstrable syllogism when, arriving at a spot where three ways meet, after smelling at two roads by which the quarry did not pass, he rushes off at once by the third without pausing to smell' (Sextus p.36). We might accept Chrysippus's claim, but it is still obvious that logic only takes off once it is crystallised in language, and even more so when an algebra is provided and precise definitions are introduced. For example, it is well-known that in ordinary talk the word 'or' comes in two version inclusive or exclusive (where the exclusive version is 'A or B, but not both'). Formal logic sorts out such ambiguities, allowing the proofs to extend indefinitely. If you add the formality of set theory to the logic (which is just hunter-gatherers thinking about groups of things) it is well known that you can launch into the higher realms of infinity, where our imagination boggles at the remoteness from ordinary experience. When have any of us ever experienced an infinity of infinities of infinities? My response to that (wearing the hat of the hardest of hard empiricists) is that the invention of formal systems is rather like the invention of the knight-move from elements of ordinary experience. Words and symbols are serious tools which can also be used as toys. Once you have seen a lizard, you can imagine a thousand of them dancing the can-can. Once you have the concept of a real mouse, you can invent Mickey Mouse. Once you have a set of tidy and accurate tools for reasoning about nature, you can do all sorts of things with your tools – but don't kid yourself that everything you do will reveal the world for us. A final check with experience will always be required. The tools of logic, though, are thoroughly empirical.

Ayer, A.J. (1936) Language Truth and Logic. Penguin. (logic as convention)

Hume, David (1748) Enquiry Concerning Human Understanding. (logic as relations of ideas)

Mill, J.S. (1843) System of Logic. (arithmetic as experienced in pebbles)

Prior, Arthur (1960) 'The Runabout Inference Ticket', in *Philosophical Logic*, ed. P.F. Strawson. OUP 1967 ('tonk' as absurd convention in logic)

Russell, Bertrand (1940) *An Inquiry into Meaning and Truth*. Penguin 1962 (connectives as psychological)

Russell, Bertrand (1912) *The Problems of Philosophy*. OUP 1980 (laws of thought as natural)

Sextus Empiricus (c. 180 CE) *Outlines of Pyrrhonism*. ed/tr. Bury,R.G. Prometheus 1990 (quote Chrysippus)

Lemmon, E.J (1965) *Beginning Logic.* Nelson 1965 (basic logic as largely natural deduction)

Bostock, David (1997) *Intermediate Logic*. OUP 1997 (Ch. 6 reduces all of logic to natural deduction)

Natural Deduction Rules

If logic is presented as 'natural deduction', you start from nothing except rules for introducing or for eliminating the various symbols of the logic. Every step of a proof can be spelled out in this way.

	introduction rule	**elimination rule**
assumption 'A'	**For the sake of argument you may assume P.**	**You may stop assuming P, if what you have proved no longer relies on P.**
and '∧', '&' or '.' [conjunction]	$\frac{P\ ,\ Q}{P\&Q}$ **If you are given P and you are given Q, you may derive their combination.** *'Moore is here; Russell is here. So Moore-and-Russell are here.'*	$\frac{P\&Q}{P}$ $\frac{P\&Q}{Q}$ **If you are given the combination of P and Q, you may derive either of them separately.** *'Moore-and-Russell are here. So Russell is here'*
or 'v' [disjunction]	$\frac{P}{PvQ}$ $\frac{Q}{PvQ}$ **If P is given, you may derive P-or-Q. If Q is given, you may derive P-or-Q.** *'Russell is here, so Russell or Moore are here'.*	$\frac{PvQ,\ P\rightarrow R,\ Q\rightarrow R}{R}$ **If P proves R and Q also proves R, and P-or-Q is given, you may derive R.** *'If Russell is here a genius is present; if Moore is here a genius is present. Either Russell or Moore are here. So a genius is present'.*
not '¬' [negation]	**If P is given and Q is proved, and not-P is given and Q is proved, you may derive Q.** *'Russell's presence means the conference is good. His absence also means the conference is good. So the conference is good'.*	**If P is given and not-P is given, then you may derive Q.** *'If Russell is here and Russell is not here, then I'll believe anything you like!'*
arrow (if-then) '→' [material implication]	**If P is given and then Q is proved, you may derive P→Q.** *'If Russell is here then Moore is here. So Russell's presence implies Moore's presence'.* [conditional proof]	**If P is given, and P→Q is given, you may derive Q.** *'Russell is here, and that implies that Moore is here. So Moore is here'.* [modus ponens]
not not '¬¬' [double negation]	**If P is given, you may derive not not-P.** *'Russell is here, so Russell is not not-here'.*	**If not not-P is given, you may derive P.** *'Russell is not not-here, so Russell is here'.*